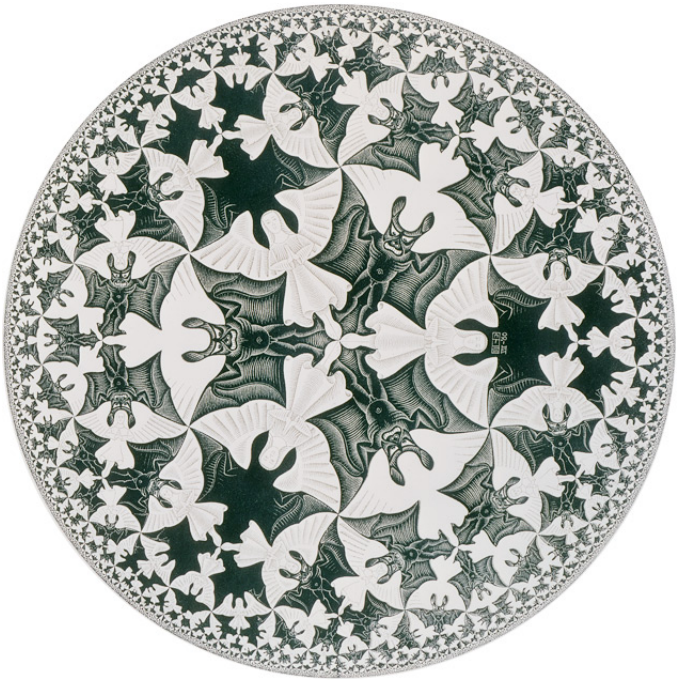
Visualization of hyperbolic surfaces



Environment

The internship is proposed by Vincent Despré and Monique Teillaud in the context of the team Gamble in the Loria laboratory in Nancy. The intern will be in contact with other members of the team including two PhD students who work on related subjects: Camille Lanuel and Loïc Dubois. Please contact Vincent Despré (vincent.despre@loria.fr) and Monique Teillaud (monique.teillaud@inria.fr) by email for further information.

Context and motivation

Traditionally, the scope of computational geometry has been limited to algorithms on data in the Euclidean space and the study of surfaces is limited to the topological aspects [dV12]. However, hyperbolic surfaces are natural and appear in various fields in physics and material sciences. For instance, the triply periodic minimal surfaces of $\mathbb{R}^3$ are hyperbolic.

The need of a geometric toolbox for hyperbolic surfaces has become critical to tackle hyperbolic surfaces. Some combinatorial constructions on such surfaces have been proposed from a mathematical point of view; however the algorithmic and practical properties have hardly been studied. We have already obtained results about the computation of Delaunay triangulations on hyperbolic surfaces [DST20, DDKT21, BDP23, DKPT23]. Thus, we now need a visualization tool to be able to see the results of our algorithms.

Objectives

Hyperbolic surfaces are hard to visualize since they cannot be embedded in $\mathbb{R}^3$. There exist different models to represent these surfaces as quotients of the hyperbolic plane, e.g., the Poincaré disk model. These models allow to produce drawings that do not exist in the Euclidean plane. For instance, there are regular polygons with arbitrary large numbers of sides that tile the hyperbolic plane, as illustrated by the painting by Escher. Actually, hyperbolic surfaces give various examples of tilings that are of particular interest. There already exist pieces of software to navigate in the hyperbolic plane but none is appropriate for the visualization of surfaces.

This subject involves both theoretical and technical aspects. Indeed, understanding and manipulating hyperbolic surfaces leads to interesting theoretical questions. On the other hand, we want to use OpenGL for the visualization, which implies technical questions. The intern will have to consider the two aspects but he/she will have some freedom to choose which one he/she wants to focus on.

Prerequisites

- Algorithmic complexity and basics of graph theory.
- Good programming skills (no knowledge of OpenGL will be assumed).
- A taste for mathematics.

References

[BDP23] Florent Balacheff, Vincent Despré, and Hugo Parlier. Systoles and diameters of hyperbolic surfaces. https://arxiv.org/pdf/2011.03455.pdf, 2023.

[DDKT21] Vincent Despré, Loïc Dubois, Benedikt Kolbe, and Monique Teillaud. Experimental analysis of Delaunay flip algorithms on genus two hyperbolic surfaces. https://hal.inria.fr/hal-03462834/document, December 2021.

[DKPT23] Vincent Despré, Benedikt Kolbe, Hugo Parlier, and Monique Teillaud. Computing a Dirichlet domain for a hyperbolic surface. https://arxiv.org/pdf/2212.01934.pdf, 2023.

[DST20] Vincent Despré, Jean-Marc Schlenker, and Monique Teillaud. Flipping geometric triangulations on hyperbolic surfaces. https://drops.dagstuhl.de/opus/volltexte/2020/12193/pdf/LIPIcs-SoCG-2020-35.pdf, 2020.

[dV12] Éric Colin de Verdière. Algorithms for graphs on surfaces. http://monge.univ-mlv.fr/~colinde/cours/algo-graphs-surfaces.pdf, 2011-2012.